SD

MA 214 **Full name:** ____________________

2/21/2024

Quiz 3

Version A **Student ID number:** 9____________________

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1. (5 points) The ODE $(2t\ln(t) - t)\,y'' + (2\ln(t) + 1)\,y' - \frac{4}{t}y = 0$ is solved by the functions $y_1 = t^2$ and $y_2 = \ln(t)$. Use the Wronskian to determine whether these two solutions form a fundamental solution set. Be sure to show your work and state your conclusion.

The Wronskian : $W(y_1, y_2)(t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} t^2 & \ln(t) \\ 2t & \frac{1}{t} \end{vmatrix}$

$= t^{\cancel{2}} \cdot \frac{1}{\cancel{t}} - 2t\ln(t) = t - 2t\ln(t)$

Note : $t - 2t\ln(t) = t(1 - 2\ln(t)) = 0 \Rightarrow \ln(t) = \frac{1}{2} \Rightarrow t = e^{1/2}$

So, the Wronskian is zero at $t = e^{1/2}$. Since $W = 0$ at one point then it is zero at every point b/c $W(y_1, y_2)(t) = W(y_1, y_2)(t_0)e^{-\int_{t_0}^{t} p(x)dx}$

Thus, $W(y_1, y_2)(t) = 0$ for all t, i.e., y_1 and y_2 do not form a fundamental solution set.

2. (6 points) Find the general solution to the following ODE.

$$y'' + 9y' - 10y = 0$$

Let $y = e^{rt} \Rightarrow y' = re^{rt}, y'' = r^2e^{rt} \Rightarrow r^2e^{rt} + 9re^{rt} - 10e^{rt} = 0$

$\Rightarrow e^{rt}(r^2 + 9r - 10) = 0 \Rightarrow r^2 + 9r - 10 = 0$

$\Rightarrow (r + 10)(r - 1) = 0 \Rightarrow r_1 = -10,\ r_2 = 1$. Then, a general solution : $y(t) = C_1 e^{-10t} + C_2 e^{t}$.

3. (9 points) The function $y_1 = \frac{1}{t}$ is a solution to the ODE

$$2t^2y'' + ty' - 3y = 0, \quad t > 0 \quad (*)$$

Use Reduction of Order to find the general solution to the ODE.

$y_1 = \frac{1}{t}$ is given to be a solution to $(*)$. Let $y_2(t) = v(t)y_1(t)$

$\Rightarrow y_2(t) = v \cdot \frac{1}{t} \Rightarrow y_2' = v' \cdot \frac{1}{t} + v \cdot \left(-\frac{1}{t^2}\right)$

$\Rightarrow y_2'' = v'' \cdot \frac{1}{t} + v' \cdot \left(-\frac{1}{t^2}\right) + v'\left(-\frac{1}{t^2}\right) + 2v \cdot \frac{1}{t^3}$

$= v'' \cdot \frac{1}{t} - 2v' \cdot \frac{1}{t^2} + 2v \cdot \frac{1}{t^3}$

Substituting y_2, y_2', y_2'' into $(*)$:

$2t^2\left(v'' \cdot \frac{1}{t} - 2v' \cdot \frac{1}{t^2} + 2v \cdot \frac{1}{t^3}\right) + t\left(v' \cdot \frac{1}{t} - v \cdot \frac{1}{t^2}\right)$

$-3v \cdot \frac{1}{t} = 0$. Expanding this

$2v''t - 4v' + \cancel{4v \cdot \frac{1}{t}} + v' - \cancel{v \cdot \frac{1}{t}} - \cancel{3v \cdot \frac{1}{t}} = 0$

$\Rightarrow 2v''t - 3v' = 0$. Reduction of order: Let $v' = w$.

$\Rightarrow 2w't - 3w = 0 \Rightarrow w' = \frac{3}{2} \cdot \frac{1}{t} \Rightarrow \frac{dw}{dt} = \frac{3}{2} \cdot \frac{1}{t}$

$\Rightarrow \int \frac{dw}{w} = \int \frac{3}{2} \cdot \frac{dt}{t} \Rightarrow \ln w = \frac{3}{2}\ln t + A = \ln t^{3/2} + A$

$\Rightarrow w = e^A t^{3/2} = At^{3/2} = v' \Rightarrow v = \int w\,dt = A\int t^{3/2} = At^{5/2} + B$

Set $A = 1$, $B = 0 \Rightarrow v = t^{5/2} \Rightarrow y_2(t) = t^{5/2}t^{-1} = t^{3/2}$

Thus, the general solution :

$$y(t) = C_1 t^{-1} + C_2 t^{3/2}$$